8: Path Integral Formulation

Saturday, April 10, 2021 8:20 PM

What is the quantum secret sauce?

Popular Science: "Superposition"

$|0\rangle - \boxed{H} - \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$

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$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\cdots$$

$$= \frac{1}{2}(|00\rangle+|01\rangle+|10\rangle+|11\rangle)\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\cdots$$

$$= \frac{1}{2\sqrt{2}}\underbrace{(|000\rangle+|001\rangle+|010\rangle+|011\rangle+|100\rangle+|101\rangle+|110\rangle+|111\rangle)}\cdots$$

With 3 qubits: Superposition of all 8-dimensional standard basis states.

Can access/explore exponentially large space!

⋮

But randomized/probabilistic computation does the same (almost)

Probabilistic Computation

Deterministic Bit: $X=0$ or $X=1$

Probabilistic Bit: $\underbrace{Pr(X=0)=.75 \quad Pr(X=1)=.25}$

Store in vector

$$\begin{pmatrix}.75\\.25\end{pmatrix}\begin{array}{l}\leftarrow Pr(0)\\ \leftarrow Pr(1)\end{array}$$

2 Probabilistic Bits?

$$\begin{pmatrix}.75\\.25\end{pmatrix} \text{ and } \begin{pmatrix}.5\\.5\end{pmatrix}$$

Bit A — Bit B

$$\begin{pmatrix}.75\\.25\end{pmatrix}\otimes\begin{pmatrix}.5\\.5\end{pmatrix}=\begin{pmatrix}3/8\\3/8\\1/8\\1/8\end{pmatrix}\begin{array}{l}\leftarrow Pr(00)\\ \leftarrow Pr(01)\\ \leftarrow Pr(10)\\ \leftarrow Pr(11)\end{array}$$

Bit A Bit B

Bits A & B

Classical Correlation:

$$\frac{1}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \qquad Pr(00)=Pr(11)=1/2$$

↖ each bit does <u>not</u> have an independent, individual state, just like entangled qubits

Quantum vs Probabilistic

Probabilistic n-bit State	Quantum n-qubit State
$\sum_{i\in\{0,1\}^n} a_i\lvert i\rangle \quad a_i\geq 0$ $\sum a_i = 1$	$\sum_{i\in\{0,1\}^n} a_i\lvert i\rangle \quad a_i\in\mathbb{C}$ $\sum \lvert a_i\rvert^2=1$
Probability of outcome i is a_i	Measure in standard basis: Probability of outcome i is $\lvert a_i\rvert^2$
Gates: Left stochastic (Preserves positivity + normalization)	Gates: Unitary (Preserves normalization)

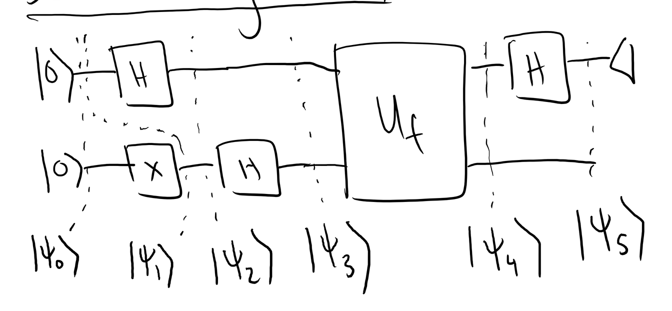
Probabilistic Gate

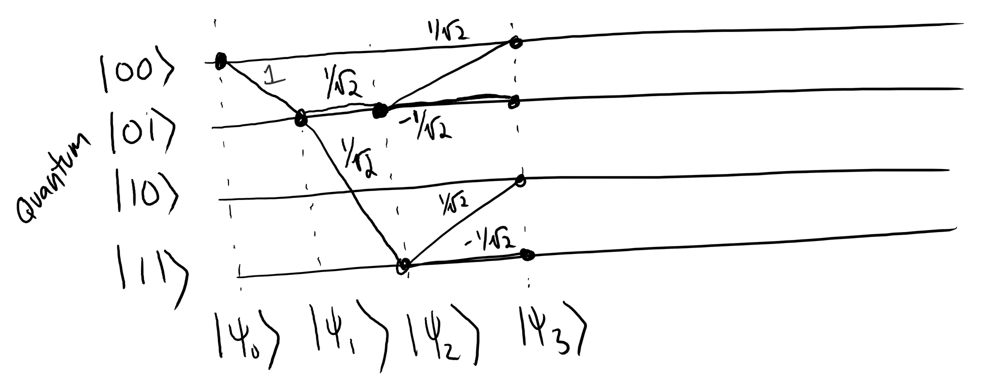
Left Stochastic Matrix (columns sum to 1, non-negative entries)

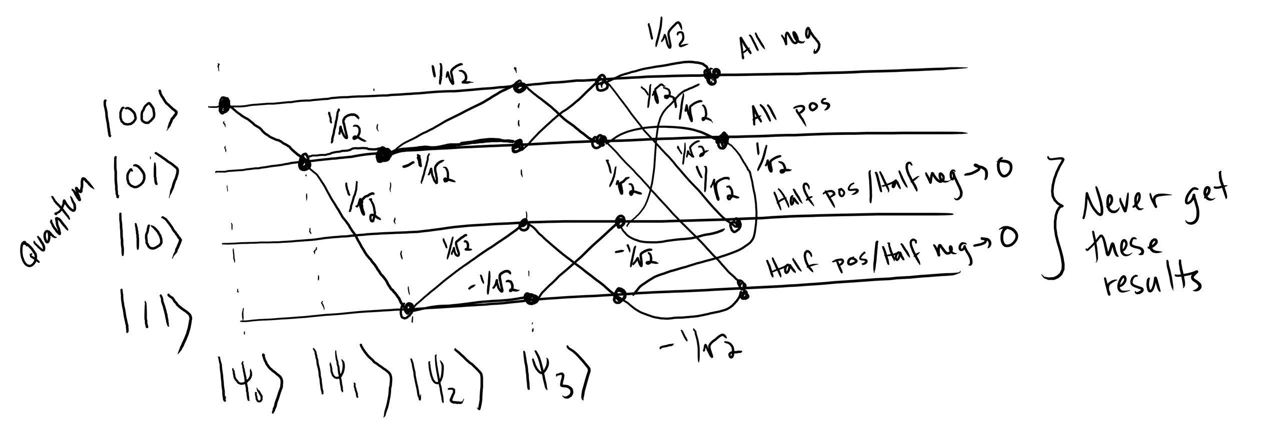
ex: $\begin{pmatrix}1/2 & 0\\1/2 & 1\end{pmatrix}\begin{pmatrix}p(0)\\p(1)\end{pmatrix}=\begin{pmatrix}1/2\,p(0)\\1/2\,p(0)+p(1)\end{pmatrix}$

Deutsch's Alg: Quantum vs. Probabilistic Paths

Deutsch Algorithm $f(0)=f(1)=1$







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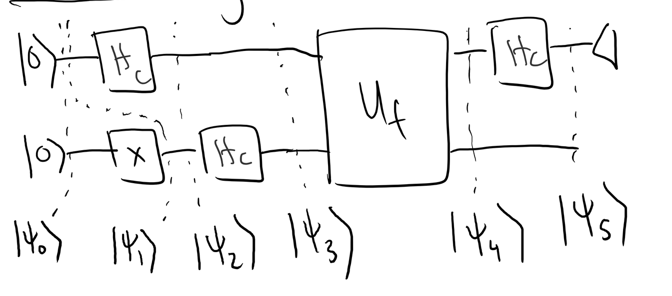
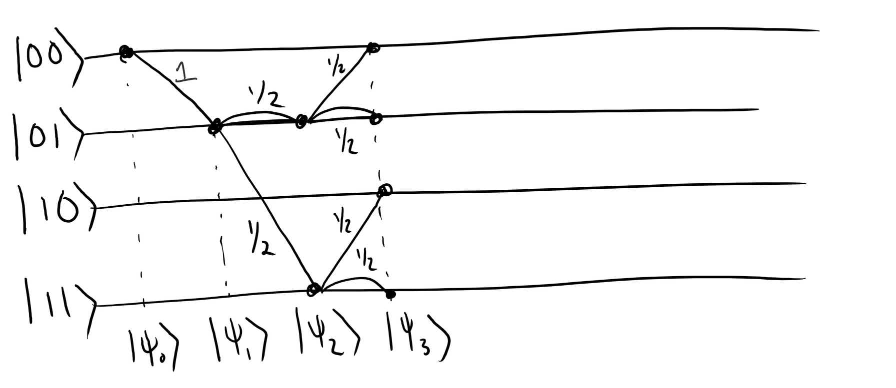
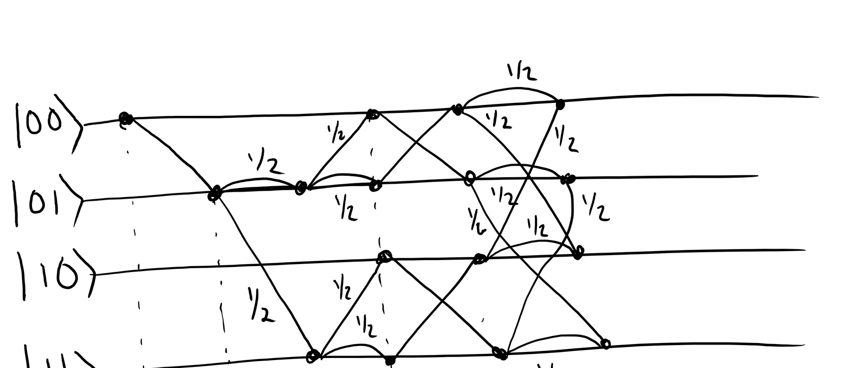


- X, U_f already left stochastic!
- H is like coin flip: $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$

  $H_c = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$





Probability of getting an outcome

- For each path, multiply numbers along path
- Add resulting product for each path that terminates at the same state
- If Quantum: abs. val. square result

Quantum Secret Sauce for Algorithms?

- Superposition + Interference

↑ pos + neg phases cancelling out bad outcomes